(D1) Calibrating the distance ladder to the LMC [75 marks]

An accurate trigonometric parallax calibration for Galactic Cepheids has long been sought, but is very difficult to achieve in practice. All known classical (Galactic) Cepheids are more than 250 pc away, therefore for direct distance estimates to achieve an uncertainty of up to 10%, parallax uncertainties of up to ±0.2 milliarcsec are needed, requiring space-based observations. The Hipparcos satellite reported parallaxes for 200 of the nearest Cepheids, but even the best of these had high uncertainties. Recent progress has come with the use of the Fine Guidance Sensor on HST where parallaxes (in many cases) accurate to better than ±10% were obtained for 10 Cepheids, spanning a range of periods from 3.7 to 35.6 days. These nearby Cepheids cover distances from about 300 to 560 pc.

The measured periods, P, and average magnitudes in V, K and I bands are given in **Table 1** as well as the A_V and A_K for extinction in V and K bands, respectively. The measured parallaxes with their uncertainties are also given in milliarcsec (mas). All measured apparent magnitudes have negligible uncertainty.

Table 1: Periods and average apparent magnitudes of 5 Galactic Cepheids with accurate parallax measurements.

	P (day)	**<V> (mag)**	**<K> (mag)**	**A_V (mag)**	**A_K (mag)**	**<I> (mag)**	**parallax (mas)**	**error (mas)**
RT Aur	3.728	5.464	3.925	0.20	0.02	4.778	2.40	0.19
FF Aql	4.471	5.372	3.465	0.64	0.08	4.510	2.81	0.18
X Sgr	7.013	4.556	2.557	0.58	0.07	3.661	3.00	0.18
ζ Gem	10.151	3.911	2.097	0.06	0.01	3.085	2.78	0.18
l Car	35.551	3.732	1.071	0.52	0.06	2.557	2.01	0.20

(D1.1) The observed correlation between the period of a Cepheid and its brightness is usually described by the so-called “Period-Luminosity (PL) relation”, where $L \propto P^{\beta}$. In fact, such a relation is normally expressed in terms of the period and absolute magnitude, instead of luminosity. Hereafter, we shall refer to the Period-Absolute magnitude relation as the conventionally named “PL relation”.

Use the data given in Table 1 to plot a suitable linear graph in order to derive the Cepheid PL relation for the V-band and K-band. You should plot each graph separately on different pieces of graph paper. Determine the slope of the line that best describes the linear relation of the data. (You may find the relation $\Delta(\log_{10} x) \approx \dfrac{\Delta x}{x \log_e 10}$ useful)

[36.5 Marks]

Any apparent differences in PL relations of stars in the different bands can be explained if one also considers differences in colour. Therefore, the PL relation is in fact a PLC (Period-Luminosity-Colour) relation. This is from the reddening effect, due to extinction being a function of wavelength, which can also vary among different Cepheids due to their different metallicities, foreground Interstellar Medium and dust.

A new reddening-free magnitude (or bandpass) called “Wesenheit” has been proposed that does not require the explicit information of the extinction of individual stars but uses colour information from the star itself to get rid of the effect. For example, W_{VI} use V and I band photometry and is defined as

$$W_{VI} = V - \left[\frac{A_V}{E(V-I)}\right](V-I),$$

$$= V - R_v(V-I)$$

where R_v depends on the reddening law. In this case, we shall take the value of R_v to be 2.45.

(D1.2) From the data given in Table 1, plot and derive the reddening-free PL relation using Wesenheit W_{VI} magnitudes. Estimate the linear slope of the relation as well as its uncertainty. [14.5 Marks]

(D1.3) Next, we would like to use the newly-derived PL relations from question (D1.1) & (D1.2) to estimate the distance to the Large Magellenic Cloud (LMC) using periods and magnitudes of classical Cepheids in the LMC. In **Table 2**, the periods, average extinction-corrected apparent magnitudes**,** $<V_{corr}>$, and Wesenheit W_{VI} magnitudes are given.

Estimate the distance modulus, μ, to each star and then use all the information to derive the distance to the LMC (in parsecs) and its standard deviation for each band.

Compare if the derived distances are statistically different for the 2 bands (YES/NO).

Are the standard deviations of the estimated distances for 2 bands different (YES/NO)?

Based on this dataset, which band (V or Wesenheit) is more accurate in estimating the distance to the LMC? [24 Marks]

Table 2: Period, average extinction-corrected apparent magnitude**,** $<V_{corr}>$, and average Wesenheit magnitude measurements of Cepheids in the LMC

	P (day)	**$<V_{corr}>$ mag**	**$<W_{VI}>$ mag**
HV12199	2.63	16.08	14.56
HV12203	2.95	15.93	14.40
HV12816	9.10	14.30	12.80
HV899	30.90	13.07	10.97
HV2257	39.36	12.86	10.54