(D2) **The search for dark matter** **[75 marks]**

A low surface brightness galaxy (LSB) is a diffuse galaxy with a surface brightness that, when viewed from the Earth, is at least one magnitude lower than the ambient night sky.

Some of its matter is in the form of "baryonic" matter such as neutral hydrogen gas and stars. However, most of its matter is in the form of invisible mass – so called "dark matter". In this question, we will investigate the mass of dark matter in a galaxy, the effect of dark matter on the rotation curves of the galaxy, and the distribution of dark matter in the galaxy.

The table below provides the data of a LSB galaxy named UGC4325. The galaxy is assumed to be edge-on. At every distance *r* from the centre of the galaxy, we measure

1. λ_{obs}, the observed wavelength of the Hα emission line. The Hubble expansion of the Universe has already been excluded from the data.

2. V_{gas}, the contribution of the gas component to the rotation due to M_{gas}, derived from HI surface densities.

3. V_*, the contribution of the stellar component to the rotation due to M_*, derived from *R*-band photometry.

The rotational velocities of the test particle due to the gas component, V_{gas}, and the star component, V_*, are defined as the velocities in the plane of the galaxy that would result from the corresponding components without any external influences. These velocities are calculated from the observed baryonic mass density distributions.

r (kpc)	λ_{obs} (nm)	V_{gas} (km/s)	V_* (km/s)
0.70	656.371	2.87	20.97
1.40	656.431	6.75	32.22
2.09	656.464	14.14	40.91
2.79	656.475	20.18	46.75
3.49	656.478	24.08	50.10
4.89	656.484	28.08	47.94
6.25	656.481	29.25	45.47
7.10	656.481	27.03	47.78
9.03	656.482	25.90	45.32
12.05	656.482	21.03	42.30

The mass of dark matter $M_{\mathrm{DM}}(r)$ within a volume of radius r can be defined in terms of the rotational velocity due to dark matter V_{DM}, the radius r and gravitational constant G,

$$M_{\mathrm{DM}}(r) = \frac{rV_{\mathrm{DM}}^2}{G}. \quad (1)$$

To a good approximation, the observed rotational velocity V_{obs} can be modelled as

$$V_{\mathrm{obs}}^2 = V_{\mathrm{gas}}^2 + V_*^2 + V_{\mathrm{DM}}^2. \quad (2)$$

The observed rotational velocity V_{obs} depends on the mass of the galaxy $M(r)$ within a volume of radius r measured from the galaxy's centre.

The mass density $\rho_{\mathrm{DM}}(r)$ of dark matter within a volume of radius r can be modelled by a galaxy density profile,

$$\rho_{\mathrm{DM}}(r) = \frac{\rho_0}{1+\left(\frac{r}{r_C}\right)^2} \quad (3)$$

where ρ_0 and r_C are the central density and the core radius of the galaxy, respectively. According to the density profile, the mass of dark matter $M_{\mathrm{DM}}(r)$ within a volume of a radius r can be described by

$$M_{\mathrm{DM}}(r) = 4\pi\rho_0 r_C^2\left[r - r_C \arctan(r/r_C)\right]. \quad (4)$$

Part 1 The mass of dark matter and rotation curves of the galaxy

(D2.1) In laboratories on Earth, Hα has an emitted wavelength λ_{emit} of 656.281 nm. Compute the observed rotational velocities of the galaxy V_{obs} and the rotational velocities due to the dark matter V_{DM} at distance r in units of km s^{-1}.

For the different values of r given in the table, compute the dynamical mass $M(r)$ and the mass of dark matter $M_{\mathrm{DM}}(r)$ in solar masses. [21]

(D2.2) Create rotation curves of the galaxy on graph paper by plotting the points of V_{obs}, V_{DM}, V_{gas}, V_* versus the radius r and draw smooth curves through the points (mark your graph as "D2.2").

Order the contribution of the different components to the observed velocity in descending order. [16]

Part 2 Dark matter distribution

(D2.3) Take a data point at small r and large r to estimate ρ_0 and r_C. Note that for large values of x, $\arctan(x) \approx \pi / 2$ and at small x, $\arctan(x) \approx x - x^3 / 3$. [7]

(D2.4) By comparing Equation (4) to a linear function, the central density ρ_0 could also be found by a linear fit. Plot an appropriate graph so that a linear fit can be used to find another value of ρ_0. Evaluate ρ_0 in units of $M_\odot$ kpc^{-3}. (Mark your graph as "D2.4"). If you cannot find the value of r_C from the previous part, use $r_C =$ 3.2 kpc as an estimate for this part. [19]

(D2.5) Compute logarithmic values of the dark matter density, $\ln[\rho_{DM}(r)]$, and plot the distribution of dark matter in the galaxy as a function of radius r on graph paper. (Mark your graph as "D2.5"). [12]