Instructions

1. The data analysis competition lasts for 5 hours and is worth a total of 150 points.

2. A dedicated IOAA **Summary Answer Sheet** is provided for writing your answers. Enter the final answers into the appropriate boxes in the **Summary Answer Sheet**. On each Answer Sheet, please fill in

- Student's Code

3. **Graph Paper** is required for your solutions. On each Graph Paper sheet, please fill in

- Student's Code
- Question no.
- Graph no. and total number of graph paper sheets used.

4. There are **Answer Sheets** for carrying out detailed work/rough work. On each Answer Sheet, please fill in

- Student's Code
- Question no.
- Page no. and total number of pages.

5. Start each problem on a separate Answer Sheet. **Please write only on the printed side of the sheet. Do not use the reverse side.** If you have written something on any sheet which you do not want to be marked, cross it out.

6. Use as many mathematical expressions as you think may help the graders to better understand your solutions. The graders may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible in English).

7. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, Graph Paper etc.), please put up your hand to signal the invigilator.

8. The beginning and end of the competition will be indicated by a long sound signal. Additionally, there will be a short sound signal fifteen minutes before the end of the competition (before the final long sound signal).

9. At the end of the competition you must stop writing immediately. Sort and put your **Summary Answer Sheet**, **Graph Papers**, and **Answer Sheets in one stack**. Put all other papers in another stack. You are not allowed to take any sheet of paper out of the examination area.

10. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition room.

11. A list of constants is given on the next page.

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity (g)	9.81 ms^{-2}	
Obliquity of Ecliptic	23°27′	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass ($M_☾$)	7.35×10^{22} kg	**Moon**
Radius ($R_☾$)	1.74×10^{6} m	
Mean Earth-Moon distance	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	−12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	**Sun**
Radius ($R_\odot$)	6.96×10^{8} m	
Luminosity ($L_\odot$)	3.83×10^{26} W	
Absolute Magnitude	4.80 mag	
Surface Temperature	5772 K	
Angular diameter at Earth	30′	
Orbital velocity in Galaxy	220 kms^{-1}	
Distance from Galactic center	8.5 kpc	
1 *au*	1.50×10^{11} m	**Physical constants**
1 *pc*	206265 au	
Gravitational constant (G)	6.67×10^{-11} Nm^{2}kg^{-2}	
Planck constant (h)	6.62×10^{-34} Js	
Boltzmann constant (k_B)	1.38×10^{-23} JK^{-1}	
Stefan-Boltzmann constant (σ)	5.67×10^{-8} Wm^{-2}K^{-4}	
Hubble constant (H_0)	67.8 kms^{-1}Mpc^{-1}	
Speed of light in vacuum (c)	299792458 ms^{-1}	
Magnetic Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ Hm^{-1}	
1 Jansky (Jy)	10^{-26} W m^{-2}Hz^{-1}	

$$\Delta \log_{10}(x) = \frac{\Delta x}{x \ln 10}$$

(D1) Dust and Young Stars in Star-forming Galaxies [75 points]

As a by-product of the star-forming process in a galaxy, interstellar dust can significantly absorb stellar light in ultraviolet (UV) and optical bands, and then re-emit in far-infrared (FIR), which corresponds to a wavelength range of 10-300 μm.

1.1. In the UV spectrum of a galaxy, the major contribution is from the light of the young stellar population generated in recent star-formation processes, thus the UV luminosity can act as a reliable tracer of the star-formation rate (SFR) of a galaxy. Since the observed UV luminosity is strongly affected by dust attenuation, extragalactic astronomers define an index called the *UV continuum slope* (β) to quantify the shape of the UV continuum:

$$f_\lambda = Q \cdot \lambda^\beta$$

where f_λ is the monochromatic flux of the galaxy at a given wavelength λ (in units of $\mathrm{W\,m^{-3}}$) and Q is a scaling constant.

(D1.1.1) (6 points) The AB magnitude is a specific magnitude system. The AB magnitude is defined as:

$$\mathrm{m_{AB}} = -2.5 \log \frac{f_\nu}{3631\ \mathrm{Jy}}$$

The AB magnitude of a typical galaxy is roughly constant in the UV band. What is the **UV continuum slope** of this sort of galaxy? (Hint: $f_\nu \Delta\nu = f_\lambda \Delta\lambda$)

(D1.1.2) (12 points) Table 1 presents the observed IR photometry results for a $z = 6.60$ galaxy called *CR7*. **Plot** the AB magnitude of *CR7* **versus the logarithm of the rest-frame** wavelength on graph paper and label it as **Figure 1**.

(D1.1.3) (5 points) Calculate *CR7*'s UV slope, **plot** the best-fit UV continuum on *Figure 1* and make a comparison with the results you obtained in (D1.1.1). Is it dustier than the typical galaxy in (D1.1.1)? **Please answer with [YES] or [NO]**. (Hint: Express $\mathrm{m_{AB}}$ as a function of λ and m_{1600} , where m_{1600} is the AB magnitude at $\lambda_0 = 160$ nm (1600 Å))

Table 1. (Observed Frame) IR Photometry of CR7 at z = 6.60

Band	*Y*	*J*	*H*	*K*
Central Wavelength (μm)	1.05	1.25	1.65	2.15
AB Magnitude	24.71 ± 0.11	24.63 ± 0.13	25.08 ± 0.14	25.15 ±0.15

1.2. Under the assumption that dust grains in the galaxy absorb the energy of UV photons and re-emit it by blackbody radiation, the relationship between the UV continuum slope (β), UV brightness (at 1600 Å) and FIR brightness can be established:

$$\mathrm{IRX} \equiv \log\left(\frac{F_{FIR}}{F_{1600}}\right) = S(\beta)$$

where F_{FIR} is the observed far-infrared flux and F_{1600} is the observed flux at rest-frame wavelength160 nm (1600 Å) (The "flux" F_λ is **defined** as $F_\lambda = \lambda \cdot f_\lambda$). Table 2 presents 20 measurements of β, F_{FIR} and F_{1600} in nearby galaxies (Meurer et al. 1999).

Table 2. UV slope, flux and FIR flux of 20 nearby galaxies

Galaxy Name	UV Slope β	$\log(F_{1600}/10^{-3}\,\mathrm{W\,m^{-2}})$	$\log(F_{FIR}/10^{-3}\,\mathrm{W\,m^{-2}})$
NGC4861	-2.46	-9.89	-9.97
Mrk 153	-2.41	-10.37	-10.92
Tol 1924-416	-2.12	-10.05	-10.17
UGC 9560	-2.02	-10.38	-10.41
NGC 3991	-1.91	-10.14	-9.80
Mrk 357	-1.80	-10.58	-10.37
Mrk 36	-1.72	-10.68	-10.94
NGC 4670	-1.65	-10.02	-9.85
NGC 3125	-1.49	-10.19	-9.64
UGC 3838	-1.41	-10.81	-10.55
NGC 7250	-1.33	-10.23	-9.77
NGC 7714	-1.23	-10.16	-9.32
NGC 3049	-1.14	-10.69	-9.84
NGC 3310	-1.05	-9.84	-8.83
NGC 2782	-0.90	-10.50	-9.33
NGC 1614	-0.76	-10.91	-8.84
NGC 6052	-0.72	-10.62	-9.48
NGC 3504	-0.56	-10.41	-8.96
NGC 4194	-0.26	-10.62	-8.99
NGC 3256	0.16	-10.32	-8.44

(D1.2.1) (14 points) Based on the data given in Table 2, **plot** the IRX $-\ \beta$ diagram on graph paper, label it as **Figure 2** and find a linear fit to the data. **Write down** your best-fit equation (i.e. $\mathrm{IRX} = a \cdot \beta + b$) by the side of your diagram.

(D1.2.2) (6 points) Quantify the **dispersion** (in 'units' of dex, where **for example**, $\mathbf{\log(10^9) - \log(10^4) = 5\ dex}$) between the observed $\mathrm{IRX_{obs}}$ and predicted $\mathrm{IRX_{pred}}$ using the following equation:

$$\sigma = \sqrt{\frac{\sum(\Delta\mathrm{IRX}_i)^2}{N-1}}\ (\mathrm{unit: dex})\ \text{where}\ \Delta\mathrm{IRX}_i = \mathrm{IRX}_{i,\mathrm{obs}} - \mathrm{IRX}_{i,\mathrm{pred}}$$

1.3. Under the previous assumption of the energy transfer process, the ratio of F_{FIR} to F_{1600} can be expressed as:

$$\frac{F_{FIR}}{F_{1600}} \approx 10^{0.4A_{1600}} - 1$$

Where F_{1600} is the unattenuated flux and A_λ is the dust absorption (in magnitudes) as a function of wavelength λ.

(D1.3.1) (6 points) Express A_{1600} as a function of IRX.

(D1.3.2) (12 points) Based on Table 2 data and the function of $A_{1600}(IRX)$ you derived above, **plot** the $A_{1600} - \beta$ diagram on graph paper, label it as **Figure 3** and find a linear fit to the data. **Write down** your best-fit equation (i.e.$A_{1600} = a' \cdot \beta + b'$) by the side of your diagram.

(D1.3.3) (2 points) If your linear model in (D1.3.2) is correct, what is the expected **UV continuum slope** β_0 of a dust-free galaxy?

1.4. After establishing the local relationship between the UV continuum slope and IRX, we can probably test this empirical law in the high-redshift universe. In 2016, researchers obtained an Atacama Large Millimeter / submillimeter Array (ALMA) observation of CR7, and the FIR continuum corresponded to a 3σ upper limit of an FIR flux of 1.5×10^{-19} W m^{-2}.

(D1.4.1) (6 points) Calculate the **IRX of *CR7***. Is it an upper limit or lower limit?

Hint: here F_{1600} should be written in the form of:

$$F_{1600} = \lambda_0 \cdot f_{1600}$$

where $\lambda_0 = 160$ nm (1600 Å) and f_{1600} is the observed flux in the rest-frame

(D1.4.2) (6 points) Is the current observation long enough to show any deviation of *CR7* from the IRX$-\beta$ relationship you just derived in the local universe? **Please answer with [YES] or [NO] on the summary answer sheet, give the IRX difference and show the working used to calculate it on the answer sheet**.

(D2) A Compact Object in a Binary System [75 points]

Astronomers discovered an extraordinary binary system in the constellation of Auriga during the course of the Apache Point Observatory Galactic Evolution Experiment (APOGEE). In these questions, you will try to analyse the data and recreate their discovery for yourself.

The research team is aiming to find compact stars in binary systems using the radial velocity (RV) technique. They examined archival APOGEE spectra of "single" stars and measured the apparent variation of their RV within this data. Among ~200 stars with the highest accelerations, researchers searched for periodic photometric variations in data from the All-Sky Automated Survey for Supernovae (ASAS-SN) that might be indicative of transits, ellipsoidal variations or starspots. After this process, they spotted a star named 2M05215658+4359220, with a large variation in RV and photometric variability.

2.1. The following table presents the radial velocity measurements of 2M05215658+4359220 during three epochs of APOGEE spectroscopic observation. Here we assume the variation of its RV is due to the existence of an unseen companion. The proper motion of the stars can be ignored.

Table 3. APOGEE Radial Velocity Measurements of 2M05215658+4359220

Observation	MJD	RV ($km\ s^{-1}$)	Uncertainty ($km\ s^{-1}$)
1	56204.9537	-37.417	0.011
2	56229.9213	34.846	0.010
3	56233.8732	42.567	0.010

(D2.1.1) (6 points) Use the data and a simple linear model to obtain an initial estimate of the apparent **maximum acceleration** of the star**:**

$$a_{max} = \left.\frac{\Delta RV}{\Delta t}\right|_{max}$$, unit: km s^{-1} day^{-1}

(D2.1.2) (9 points) Now use the data to obtain an initial estimate of the **mass** of its unseen companion.

2.2. After discovering this peculiar star, astronomers conducted follow-up observations using the 1.5-m Tillinghast Reflector Echelle Spectrograph (TRES) at the Fred Lawrence Whipple Observatory (FLWO) located on Mt. Hopkins in Arizona, USA. The following table presents the RV measurements using this instrument:

Table 4. TRES Radial Velocity Measurements of 2M05215658+4359220

MJD	RV ($km\ s^{-1}$)	Uncertainty ($km\ s^{-1}$)
58006.9760	0	0.075
58023.9823	-43.313	0.075
58039.9004	-27.963	0.045
58051.9851	10.928	0.118
58070.9964	43.782	0.075
58099.8073	-30.033	0.054
58106.9178	-42.872	0.135
58112.8188	-44.863	0.088
58123.7971	-25.810	0.115
58136.6004	15.691	0.146
58143.7844	34.281	0.087

(D2.2.1) (14 points) **Plot** the diagram of RV variation (measured with TRES) versus time on your graph paper and label it as Figure 4. Draw a suitable **sinusoidal function** to fit the given data. **Estimate** the orbital period (P_{orb}) and radial velocity amplitude (K) from your plot.

(D2.2.2) (4 points) If the star is moving in a circular orbit, calculate the **minimum value** of the orbital radius (r_{orb}) of the star in units of both $R_\odot$ and au.

(D2.2.3) (7 points) The mass function of a binary system is defined as:

$$f(M_1, M_2) = \frac{(M_2 \sin i_{orb})^3}{(M_1 + M_2)^2}$$

where the subscript "1" represents the primary star and "2" represents its companion. The parameter i_{orb} is the orbital inclination of the binary system. This mass function can also be expressed in terms of observable parameters. Calculate the **mass function of this system** in units of $M_\odot$.

2.3. Based on a detailed analysis on APOGEE, TRES spectra and GAIA parallax measurements, astronomers derived the following stellar parameters:

Table 5. Selected Physical Properties of 2M05215658+4359220

Effective Temperature $T_{\rm eff}$ (K)	Surface Gravity $\log g$ (cm s^{-2})	Parallax π (mas)	Measured Rotation Velocity $v_{rot} \sin i$ (km s^{-1})	Bolometric Flux F (W m^{-2})
4890 ± 130	2.2 ± 0.1	0.272±0.049	14.1±0.6	(1.1±0.1)×10^{-12}

Photometric observations indicate that the period of its light curve is identical to its orbital period, thus we may assume that the rotation period satisfies $P_{rot} = P_{orb} \equiv P$, and the inclination satisfies $i_{orb} = i_{rot} \equiv i$.

(D2.3.1) (16 points) **Calculate** the luminosity (L_1, in units of $L_\odot$), radius (R_1, in units of $R_\odot$), and sine of the inclination angle ($\sin i$), as well as the mass (M_1, in units of $M_\odot$) of the visible star. Please **include** the uncertainty in your results.

(D2.3.2) (4 points) **Choose** the correct type of this star from the following options: (1) Blue Giant (2) Yellow main sequence star (3) Red Giant (4) Red main sequence star (5) White Dwarf.

(D2.3.3) (10 points) Based on the mass function $f(M_1, M_2)$ of the binary system, **plot** the rough relationship of M_2 (on the vertical axis) and M_1 (on the horizontal axis) on your graph paper and label it as Figure 5. Plot the most probable relationship (by using $\sin i$), the upper limit on the relationship (with $\sin i + \Delta \sin i$) and the lower limit (with $\sin i - \Delta \sin i$) as derived in (D2.3.1).

(D2.3.4) (5 points) Draw a vertical **shaded region** of $[M_1 - \Delta M_1, M_1 + \Delta M_1]$, as well as two horizontal **dashed lines** showing the maximum mass of the white dwarf and neutron star, on your *Figure 5*. What is the possible mass of the invisible companion, and what kind of celestial object could it be?