The 4th IOAA
Theoretical Competition
Short Problem

Please read these instructions carefully:

1. Each student will receive problem sheets in English and/or in his/her native language.
2. The time available for answering theoretical problems is 5 hours. You will have 15 short problems (Theoretical Part 1, Problem 1 to 15), and 2 long problems (Theoretical Part 2, Problem 16 and 17).
3. Use only the pen that has been provided on your desk.
4. Write your answer to each problem on a new page in the notebook.
5. Begin answering each problem in separate sheet. Write down the number of the problem at the beginning.
6. Write down your "country name", your "student code" on the cover of the notebook.
7. The final answer in each question part must be accompanied by units and must be of correct number of significant digits (use SI or appropriate units). 20% of the marks available for that part will be deducted for a correct answer without units.
8. At the end of the exam put all papers and the notebook inside the envelope and leave everything on your desk.

1) Visual binary: the primary's magnitude is 1.0, the second's magnitude is 2.0, please calculate the magnitude of the binary system.

In a binary system, the apparent magnitude of the primary star is 1,0 and that of the secondary star is 2.0. Find the maximum combined magnitude of this system.

Solution:

Let F_1, F_2, and F_0 be the flux of the first, the second and the binary system, respectively.

$$\Delta m = -2.5\lg(F_1 / F_2)$$
$$(1-2) = -2.5\lg(F_1 / F_2)$$ 5

So, $F_1 / F_2 = 10^{1/2.5} = 10^{0.4}$

$$F_0 = F_1 + F_2 = F_1(1+10^{-0.4})$$ 3

The magnitude of the binary m is:

$$m - 1 = -2.5\lg(F_0 / F_1) = -2.5\lg(F_1(1+0.398)/F_1) = -0.36^m$$ 2

So, $m = 0.64^m$

2) If the escape velocity from a solar mass object's surface exceeds the speed of light, what would be its radius ?

Solution:

$$\because \sqrt{\frac{2Gm_{sun}}{R}} > c$$ 4

$$\therefore R_{sun} = R < 2Gm_{sun} / c^2$$ 3

So, $R_{sun} < 2\times 6.67\times 10^{-11}\times 1.99\times 10^{30} / (3\times 10^8)^2 m \approx 2950m$ 3

3) A newly-discovered unknown object located at a distance of 50 pc appears as bright as the Sun. Estimate its luminosity (in terms of luminosity of the Sun).

Original revised:

The redshift of a QSO is $z = 0.20$, estimate the distance of the QSO. The Hubble constant is 72 km s^{-1} Mpc^{-1}.

Solution:

Recession velocity of the QSO is

$$\frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} = 0.156$$ 2

According to the Hubble's law,

$$v = H_0 D$$

The distance of the QSO is

$$D = v / H_0 = 0.156c / 72 = 650Mpc,$$ 2

4) A binary system is 10 pc away, the largest angular distance between the components is 7.0", the smallest value is 1.0", the orbital period is 100 years, assuming the orbital plane is perpendicular to the line of sight. If the semi-major axis of one component corresponds to 3.0", that is $a_1 = 3''$, estimate the mass of each component of the binary system, in terms of solar mass.

Solution:

The semi-major axis is

$$a = 1/2 \times (7+1) \times 10 = 40AU$$ 2

From Kepler's 3rd law,

$$M_1 + M_2 = \frac{a^3}{p^2} = \frac{(40)^3}{(100)^2} = 6.4M_{sun}$$

4

since $a_1 = 3'', a_2 = 1''$,then

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$ 2

$$m_1 = 1.6m_{sun}, m_2 = 4.8m_{sun}$$ 2

5) If 0.8% of the initial total solar mass could be transformed into energy during the whole life of the sun, estimate the maximum possible life time for the sun? Assume that the solar luminosity remains constant.

Solution:

The total mass of the sun is

$$m \approx 2 \times 10^{30} kg$$ 1

0.8% mass transform into energy:

$$E = mc^2 \approx 0.008 \times 2 \times 10^{30} \times (3 \times 10^8)^2 = 1.4 \times 10^{45} J$$ 4

Luminosity of the sun is

$$L_{sun} = 3.9 \times 10^{26} W$$ 1

Sun's life would at most be:

$$t = E / L_{sun} = 3.6 \times 10^{18} s \approx 10^{11} years$$ 4

6) A spacecraft landed on the surface of an asteroid with negligible rotation, whose diameter is 2.2

km, and its average density is 2.2g/cm^3. Can the astronaut complete a circle along the equator of the asteroid on foot within 2.2 hours? Write your answer "YES" or "NO" on the answer sheet and explain why with formulae and numbers.

Solution:

From energetic formula,

$$v^2 = \mu(\frac{2}{r} - \frac{1}{a})$$

$$\mu = G(m_1 + m_2)$$

The mass of the asteroid is

$$m_1 = \frac{4}{3}\pi r^3 \rho = 1.23\times 10^{13} kg$$ 2

Since $m_2 << m_1$, m_2 can be omitted,

Then $v = \sqrt{\frac{Gm_1}{r}} = 0.862 km/s$ 3

It is the first cosmological velocity of the asteroid.

If the velocity of the astronaut is greater then v, he will escape from the asteroid.

The astronaut must be at v_2 if he wants to complete a circle along the equator of the asteroid on foot within 2.2 hours, and

$$v_2 = \frac{2\pi\times(2200/2)}{(2.2\times 3600)} = 0.873 km/s$$ 3

Obviously $v_2 > v$

So the answer should be "NO". 2

If our Sun were suddenly replaced with a black hole of the same mass, re-calculate the Earth's orbit (around the black hole). Can we continue living on Earth? Please write your answer "YES" or "NO" on the answer sheet.

Solution:

The earth's orbit around the black hole would remain. 4

We can't live on the earth, so the answer should be "NO". 4

the Earth's temperature would change, and there would be no solar wind or solar magnetic storms affecting us. 2

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7) We are interested in finding habitable exoplanets. One way to achieve this is through the dimming of the star, when the exoplanet transits across the stellar disk and blocks a fraction of the light. Estimate the maximum flux change ratio for an Earth-like planet orbiting a star similar to the sun.

Solution：

The flux change is proportional to the ratio of their surface areas, i.e.,

$$(R_e / R_{sun})^2 = 7.4\times10^{-5}$$

Obviously this difference is extremely small.

8) The Galactic Center is believed to contain a super-massive black hole with a mass M=4 $\times10^6$ $M_\odot$. The astronomy community is trying to resolve its event horizon, which is a challenging task. For a non-rotating black hole, this is the Schwarzschild radius, $R_s = 3(M/M_\odot)$ km. Assume that we have an Earth-sized telescope (using Very Long Baseline Interferometry). What wavelengths should we adopt in order to resolve the event horizon of the black hole? The Sun is located at 8.5 kpc from the Galactic Center.

Solution:

Observationally, the diameter of the Galactic black hole at the distance of $L = 8.5kpc$ has the angular size,

$$\theta_{BH} = 2R_s / L \qquad 2$$

On the other hand, an Earth-sized telescope ($D = 2R_e$) has the resolution,

$$\theta_{tel} = 1.22\lambda/(2R_e) \qquad 2$$

In order to resolve the black hole at Galactic center, we need to have $\theta_{BH} \geq \theta_{tel}$,which marginally we consider $\theta_{BH} = \theta_{tel}$

This leads to,

$$\lambda = R_e R_s / L / 0.3 \qquad 4$$

Taking the values, we have (assuming R_e = 6000 km),

$$\lambda = 0.09cm \qquad 2$$

This means that we need to observe at least at near sub-mm frequencies, which is in radio or far-infrared band.

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7) A star has a measured I-band magnitude of 22.0. How many photons per second are detected from this star by the Gemini Telescope(8m diameter),assuming that the overall quantum efficiency is 40%?

You can use the following information, for Vega, which has an I-band magnitude of $m_I = 0.0$

$Filter$	$\lambda_{eff}(nm)$	$\Delta\lambda(nm)$	$F(Wm^{-2}nm^{-1})$
I	800	240	0.83×10^{-11}

Solution:

The definition of the magnitude is:

$$m_I = -2.5\lg F_I + const \qquad 1$$

Where F_I is the flux received from the source. Using the data above, we can obtain the constant:

$$0.0 = -2.5\lg(0.83\times10^{11}) + const$$
$$const = -27.7 \qquad 1$$

Thus,

$$m_I = -2.5\lg F_I - 27.7$$
$$F_I = 10^{\frac{m_I+27.7}{-2.5}} = 1.3\times10^{-20} Wm^{-2}nm^{-1} \qquad 2$$

For our star, at an effective wavelength $\lambda_{eff} = 800nm$

using this flux, the number of photons detected per unit wavelength per unit area is the flux divided by the energy of a photon with the effective wavelength:

$$N_I = \frac{1.3\times10^{-20}}{hc/\lambda_{eff}} = 5.3\times10^{-2}\, photonss^{-1}m^{-2}nm^{-1} \qquad 3$$

Thus the total number of photons detected from the star per second by the 8m VLT UT over the I band is

$$N_I(total) = (tel.collectingarea)\times QE\times Bandwidth\times N_I$$
$$= (\pi\times4^2)\times0.4\times240\times N_I \qquad 3$$
$$= 255\, photonss^{-1}$$

8) Assuming that the G-type main-sequence stars(such as the Sun)in the disc of the Milky Way obey a vertical exponential density profile with a scale height of 300pc, by what factor does the density of these stars change at 0.5 and 1.5kpc from the mid-plane reletive to the density in the mid-plane?

Solution:

Since $h_z = 300pc$, we can substitute this into the vertical(exponential)disc equation:

$$n(0.5kpc) = n_0\exp(-|500pc|/300pc) \approx 0.189n_0 \qquad 5$$

In other words, the density of G-type MS stars at 0.5kpc above the plane is just under 19% of its mid-plane value.

For $z = 1.5kpc$,this works out as 0.007. 5

9) The Mars arrived at its great opposite at UT 17^h56^m Aug.28, 2003. The next great opposite of the Mars will be in 2018, please estimate the date of that opposite.

Solution:

$$\frac{1}{T_s}=\frac{1}{T_E}-\frac{1}{T_M}$$ 4

$$T_s=\frac{T_E\times T_M}{(T_M-T_E)}=779.95days$$

That means there is an opposite of the Mars about every 780 days.

If the next great opposite will be in 2018, then

$$15\times365+4=5479days$$ 2

$$5479/779.95=7.025$$ 1

It means that there will have been 7 opposites before Aug.28, 2018,

So the date for the great opposite should be

$5479-7\times779.95=19.35days$, i.e.

19.35days before Aug.28,2018, 3

It is on Aug .9, 2018.

10) The light variation of a Cepheid is 2^m,if its effective temperature at maximum luminosity is 6000K, while at minimum is 5000K, please estimate the ratio of its maximum and minimum radius.

Solution:

$$L_{\max}=4\pi R_{\max}^2\sigma T_{\max}^4$$ 3

$$L_{\min}=4\pi R_{\min}^2\sigma T_{\min}^4$$

$$\Delta m=-2.5\lg(L_{\min}/L_{\max})=-5\lg(R_{\min}/R_{\max})-10\lg(T_{\min}/T_{\max})$$ 3

$$\lg(R_{\min}/R_{\max})=-0.2\Delta m-2\lg(T_{\min}/T_{\max})=-0.24$$ 2

So,

$$R_{\min}/R_{\max}=0.57$$ 2

11) Please use Wien's displacement law to estimate the effective temperature of our Sun.

Solution:

The Wien law is

$$\lambda_{\max}=\frac{0.29}{T}(cm)$$ 5

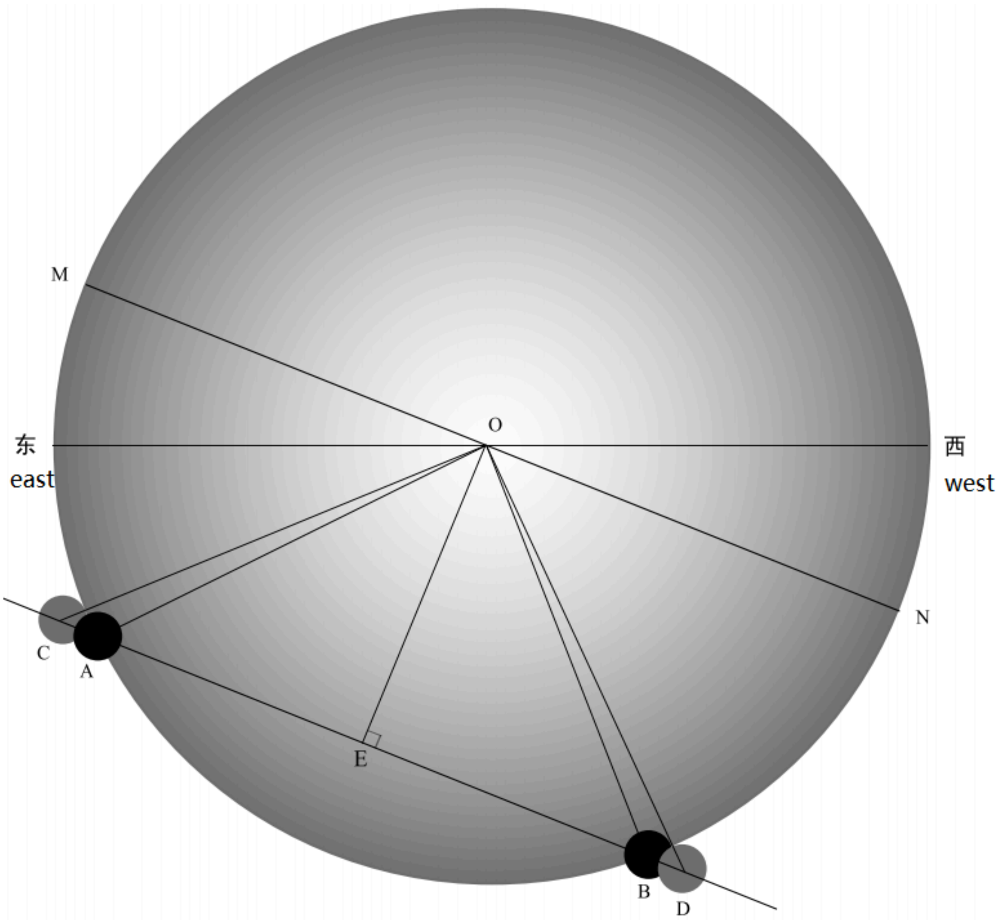
So the temperature is

$$T=\frac{0.29}{550\times10^{-9}}=5272K$$

Or

$$T = \frac{0.29}{500\times10^{-9}} = 5800K$$ 5

12) An observer observed a transit of Venus at the north pole of the Earth. The transit path of Venus is shown as the picture(A,B,C,D are all on the path of transit and marking the center of the Venus disk; at A and B, the center of the Venus superposed on the limb of the Sun disk; C corresponds to the first contact while D the fourth contact , $\angle AOB = 90^\circ, MN \| AB$).The first contact began from 9^h LMT(local mean time), please calculate on what time the transit finished.



Solution:

Since the observer is at the pole, the affect of the earth's rotation on the transit could be neglected.

$T_{venus} = 224.70 days$、$T_{earth} = 365.25 days$; $a_{venus} = 0.723 AU$

The radius of the Venus $r = 0.949\ r_{earth} = 6052.72 km$;

the solar radius $r_{sun} = 6.96\times10^{8} m$;

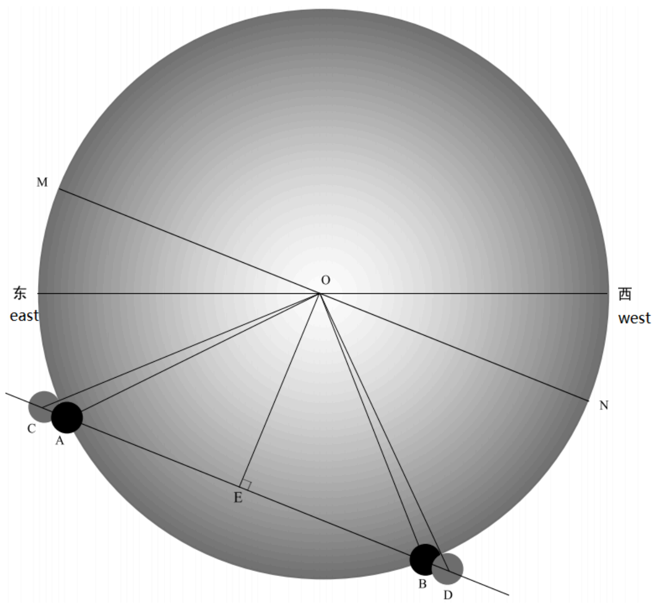
then the sun's angle at the earth extends as $\theta_0 = \arcsin(\frac{2r_{sun}}{1AU}) \approx 32.0'$;

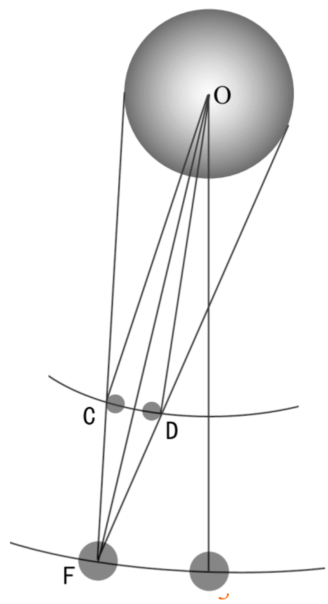
the angular velocity of the Venus around the sun, respected to the earth is ω_1,

$$\omega_1 = \omega_{venus} - \omega_{earth} = \frac{2\pi}{T_{venus}} - \frac{2\pi}{T_{earth}} \approx 4.29\times10^{-4}('/s) \qquad 2$$

For the observer on earth, Venus moved θ during the whole transit ,
Let OE be perpendicular to AB,
OA=16′ □AOB=90°, MN□AB,

So $OE = 11.3'$, $OC = \frac{d'_{venus}}{2} + \frac{r'_{sun}}{2}$, d'_{venus} is the angular size of Venus seen from Earth.





$$d'_{venus} = \frac{2\times0.949\times6378}{(1-0.723)\times1AU} \approx 1',$$

$$OC \approx 16.5', \quad CD \approx 24.0',$$

$$CE = \sqrt{OC^2 - OE^2} \approx 12.0'$$

$$CD = 2CE = 24.0'$$

So, $\theta = \angle CFD = 24.0'$,

As shown on the picture,

$\theta' = \angle COD$ is the additional angle that Venus covered during the transit,

$$\frac{tg\frac{\theta}{2}}{tg\frac{\theta'}{2}} = \frac{0.723}{(1-0.723)}, \quad tg\frac{\theta}{2} = tg12', \theta' = 9.195'; \qquad 3$$

$$t_{transit} = \frac{\theta'}{\omega_1} = \frac{9.195'}{4.29\times10^{-4}{}' / s}\times\cos\varepsilon$$, that is $5^h56^m36^s$,

So the transit will finish at about 14^h57^m. 2